

The Stieltjes and Hausdorff Moment Problems

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Abstract

We discuss classical moment problems, focusing on the Stieltjes and Hausdorff cases. Given a sequence of real numbers, the central question is whether it can be represented as the moments of a positive measure supported on a prescribed domain. In the Stieltjes moment problem, the measure is supported on $[0, \infty)$, while in the Hausdorff moment problem, the support is restricted to $[0, 1]$. We present fundamental characterizations of such sequences in terms of positivity of Hankel matrices and complete monotonicity conditions, respectively. Particular emphasis is placed on the contrast between the two problems, especially regarding the existence and uniqueness of representing measures. We also illustrate these ideas through examples and briefly indicate connections with functional analysis and operator theory.